Pointing Error in Passively Stabilized Spacecraft Caused by Thermal Bending

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An important component of passively stabilized earth orbiting spacecraft is a long thin-walled cylinder that points at all times towards the earth. When the cylinder is exposed to the sun, the side facing the sun heats up more than the far side causing it to bend away from the sun. In general, the differential thermal expansion produces stresses in the material of the boom; however, there is a particular distribution that does not produce stresses. Equations for determining the bending radius are derived for the no-stress and the general cases. Sample calculations are presented; although the results obtained for the two cases do not, in general, differ very much, there are instances for which the discrepancy is significant.

Nomenclature

a = absorptivity
A = area of boom cross section, in.^{2}
d = boom diameter, in.
e = external emissivity
e_i = internal emissivity
E = Young's modulus of elasticity, psi
I = moment of inertia, in.^{4}
k = thermal conductivity, Btu-ft/hr-ft^2-°F
K = integer
m = number of sides of equilateral polygon
M = couple, lb-in.
P = force, lb
R = radius of curvature, in.
R_L = radius of curvature according to the linearized theory, in.
R_i = projection of the distance between center fiber and edge of the ith element on the axis b-b (Fig. 4), in.
s = boom thickness, in.
t = temperature, °F
t_{max} = maximum temperature, °F
t_{min} = minimum temperature, °F
Δt = $t_{max} - t_{min}$
y = distance from center fiber, in.
α = coefficient of thermal expansion, in./in.-°F
ϵ = unit elongation
θ = angle defined in Fig. 1

Introduction

RECENTLY, a satellite[1] was launched to demonstrate that space vehicles can be passively stabilized to point at all times in the direction of the local vertical using the gravity gradient effect. An important component of the system is a 100-ft-long, thin-walled tube (henceforth called the boom) that extends after the satellite is in orbit. The successful experiment showed that the uneven temperature distribution on the periphery of the boom induced by solar radiation is a cause of pointing error.

Steady-state temperature distribution in a boom takes place when the heat received is equal to the amount reradiated into space. It is determined by the thermal properties of the surfaces, conductivity of the material, and the geometry of the boom configuration. The distribution can be obtained by analog[2] and digital techniques by subdividing the boom into sufficiently large number of sections and considering the heat balance of each section. The full line in Fig. 1 shows the temperature profile of an infinitely long, 2-in.-diam beryllium copper boom 0.002 in. thick in near-earth orbit, where $a/e = 1$, $a = 0.05$, and $e_i = 0.85$ as given in Ref. 2. Charnes and Raynor[3] obtained an approximate solution of the nonlinear, differential equation governing the temperature profile, neglecting internal and earth radiation. The left side of Fig. 2 shows the relation of the maximum temperature difference and the product of boom thicknesses and conductivity for various boom diameters. It is assumed that the surface of the boom is highly polished and suitably coated ($a = 0.05$, $e = 0.15$), representing the present state of the art. The radiation effects increase with time because of the surface degradation in space. For angles of incidence other than 90°, the difference between the temperature at any point on the periphery and the average temperature diminishes proportionately with the $\frac{1}{4}$ power of the cosine of the angle of incidence of the solar radiation. Increase of the angle of incidence also reduces the average temperature, but this in itself does not influence boom bending since it does not induce differential elongation. In general, the temperature profile induces internal stresses in the boom. The particular temperature distribution that does not induce stresses is compared with the actual distribution in Fig. 1. We will discuss boom bending caused by the no-stress and the general cases separately.

Bending Due to No-Stress Temperature Profile

The no-stress peripheral temperature profile was chosen to yield temperatures proportional to the distance y from the central fibers[4] of the boom:

$$t(\theta) = (t_{max} + t_{min})/2 + (t_{max} - t_{min})y/d \qquad (1)$$

where $y = (d/2)\cos\theta$.

The radius of curvature R_L of the bent boom is determined from the similarity of the triangles $0ab$ and bcd in

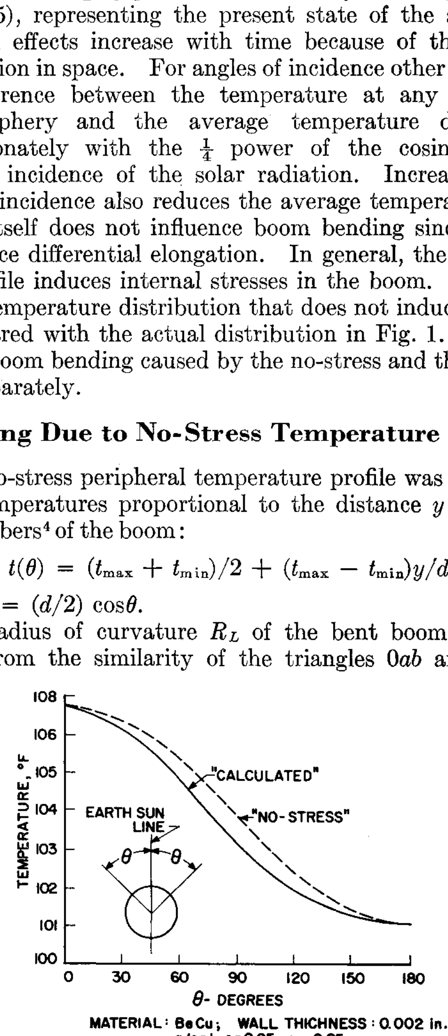

Fig. 1 Temperature distribution on the periphery of a 2-in.-diam boom.

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Fig. 3a. The elongation corresponding to the cross section *e-e* is given in Fig. 3b. The unit elongation is $\epsilon = y/R_L = cd/ab = \alpha\Delta t y/d$. The radius of curvature is given by

$$R_L = \Delta t d/\alpha \tag{2}$$

Figure 2 is a combined diagram of the temperature difference Δt as given in Ref. 3 and Eq. (2) for various boom diameters and for material properties of interest. The radius of curvature is obtained by following the path A, B, C, and D of the sample calculation of a 2-in.-diam beryllium copper boom, 0.002 in. thick, having

$$sk = 0.112 \left[\text{in.}\,\frac{\text{Btu-ft}}{\text{hr-°F-ft}^2}\right]$$

$$\frac{d}{\alpha} = 0.02 \left[\frac{\text{in.}}{\text{in./in.-°F}}\right]$$

The resulting maximum temperature differential and radius of curvature are $\Delta t = 5.5$°F and $R_L = 4200$ ft. A boom of the same geometry but made of stainless steel, having

$$sk = 0.024 \left[\text{in.}\,\frac{\text{Btu-ft}}{\text{hr-°F-ft}^2}\right]$$

$$\frac{d}{\alpha} = 0.167 \left[\frac{\text{in.}}{\text{in./in.-°F}}\right]$$

results in $\Delta t = 27$°F and $R_L = 600$ ft. The part of the boom facing the sun expands more than the unexposed side, causing it to bend away from the sun.

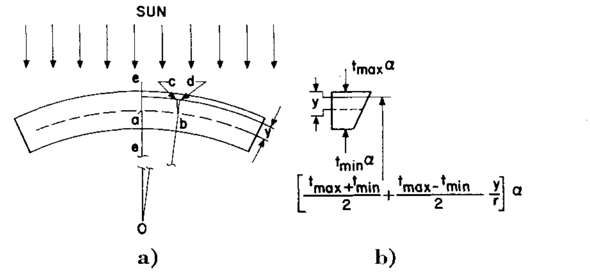


Fig. 3 a) Boom bending caused by the linearized temperature profile; b) elongation of an arbitrary cross section *e-e*.

Bending Due to Arbitrary Temperature Profile

Let us approximate the circular cross section of the boom with an equilateral polygon (Fig. 4) of $m = 4(K + 1)$ sides, where K is an integer. The elements are labeled 0, 1, . . . $m/2$ in either direction starting with the element facing the sun. Each element is considered to be of uniform temperature corresponding to the temperature of its c.g.

If the elements were not rigidly connected, they would expand axially, proportionally to the imposed temperature profile. However, because of the constraint, they are subjected to bending and tension or compression. The internal forces in each element can be reduced to an axial force P_i acting at the c.g. of the element and a couple M_i, so that

$$\sum_{i=0}^{m/2} P_i = 0 \tag{3}$$

$$\frac{d}{2}\sum_{i=0}^{m/2} P_i\left[1 - \cos\left(\frac{2i\pi}{m}\right)\right] + \sum_{i=0}^{m/2} M_i = 0 \tag{4}$$

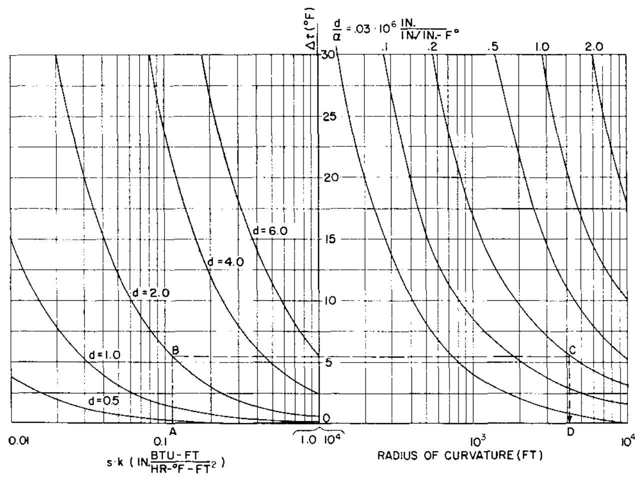


Fig. 2 Radius of curvature of booms based on linearized theory.

where the bending moments were taken about the axis *a-a* that goes through the c.g. of element 0. There are no resultant forces or bending moments acting perpendicular to axis *a-a* because of the symmetry of the cross section of the boom and of the temperature profile.

Assuming that $d \ll R_L$ and $s \ll d$, the relation among the bending moment, flexural rigidity, and radius of curvature for each element is given by

$$M_i = EI_i/R \qquad i = 0, 1, 2, \ldots m/2 \tag{5}$$

where

$$I_i = (s/6)(d\pi/m)^3 \sin^2(2i\pi/m) \tag{6}$$

Substituting Eqs. (5) and (6) into (4), we eliminate M_i and get

$$\sum_{i=0}^{m/2}\left[P_i\left(1 - \cos\frac{2i\pi}{m}\right)\frac{d}{2} + \frac{Es}{6R}\left(\frac{d\pi}{m}\right)^3 \sin^2\frac{2i\pi}{m}\right] = 0 \tag{7}$$

Another relation is derived from the condition that the unit enlongation is the same at both sides of the interfaces of the elements. Equating the elongations of the adjacent edges of the ith element and of the $(i + 1)$ element, we have

$$\alpha t_i + P_i/EA_i - \Delta R_i/R = \alpha t_{i+1} + P_{i+1}/EA_{i+1} + \Delta R_{i+1}/R \tag{8}$$

The first term on either side of Eq. (8) is caused by the thermal elongation. It is assumed throughout this report that the coefficient of thermal expansion is constant throughout the boom. The second terms are the unit elongations

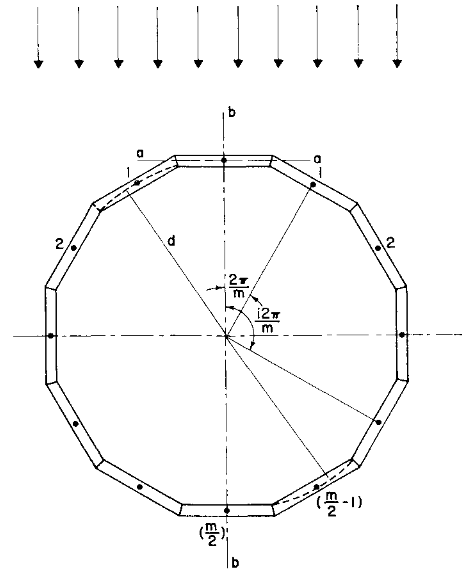
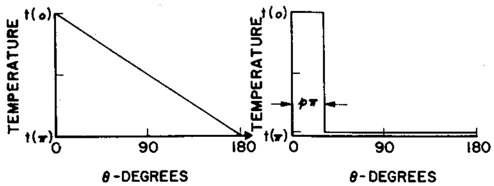


Fig. 4 Polygon approximation of circular cross section.



a) Linear distribution b) Step distribution

Fig. 5 Temperature distribution for which β can be determined in closed form.

caused by the longitudinal force P_i, and the third terms are caused by the curvature of the boom.

From the geometry of Fig. 4, we have $\Delta R_i = d\pi/2m \sin(2i\pi/m)$. Substituting into Eq. (8), we get

$$(P_i - P_{i+1})/EA - d\pi/2mR[\sin(2i\pi/m) + \sin 2\pi(i+1)/m] = \alpha(t_{i+1} - t_i) \qquad i = 0, 1, 2, \ldots (m/2 - 1) \quad (9)$$

Equations (3, 7, and 9) are a system of $(m/2) + 2$ linear equations with the unknowns $P_0, P_1, P_2, \ldots P_{m/2}$, and R. We rewrite the equations by letting the number of sides of the polygon go to infinity, multiplying and dividing by $d\theta = 2\pi/m$, and substituting trigonometric relations to get

$$\int_0^\pi P(\theta)\, d\theta = 0 \qquad \int_0^\pi P(\theta)(1 - \cos\theta) d\theta = 0 \quad (10)$$

Hence,

$$\int_0^\pi P(\theta) \cos\theta \, d\theta = 0 \quad (11)$$

$$(dP/d\theta) + (EA/2R) \sin\theta = -EA dt/d\theta \quad (12)$$

where $A = \pi d^2/4$. Integrating (12) from 0 to θ,

$$-[P(\theta) - P(0)] + (EA/2R)(\cos\theta - 1) = EA\alpha[t(\theta) - t(0)] \quad (13)$$

Now if (13) is multiplied by $\cos\theta$ and integrated from 0 to π, using (11) and

$$\int_0^\pi \cos\theta \, d\theta = 0 \qquad \int_0^\pi \cos^2\theta \, d\theta = \frac{\pi}{2}$$

we obtain for the radius of curvature of the boom

$$R = \frac{\pi}{4}\left[\int_0^\pi \frac{t(\theta)}{\Delta t} \cos\theta \, d\theta\right]^{-1} \frac{d}{\alpha \Delta t} = \beta R_L \quad (14)$$

Equation (14) is expressed in terms of the radius of curvature R_L, derived by the use of the linearized temperature profile [Eq. (2)], and multiplied by the "temperature profile factor" β. The temperature profile appears in β in normalized form. Substituting for the temperature profile $t(\theta)$, the linearized profile given by Eq. (1), we obtain $\beta = 1$ as it should be.

Sample Calculations and Conclusions

The bending of a boom induced by an arbitrary temperature profile is independent of the modulus of elasticity and thickness of the material of the boom. The radius of curvature is proportional to the boom diameter, and the temperature profile factor is inversely proportional to the coefficient of expansion of the boom material and the maximum temperature difference. The derivation of Eq. (14) is limited by the assumptions that the boom diameter is much smaller than the radius of curvature and the boom thickness is much smaller than the boom diameter.

The numerical evaluation of the bending radius is accomplished in two steps: 1) determine the radius of curvature R_L caused by the linearized temperature profile, and 2) determine β, the temperature profile factor. R_L is plotted for a wide range of conditions in the diagram in Fig. 2. The temperature profile factor must be determined separately for each given profile by evaluating the term containing the integral in Eq. (14). For example, the radius of curvature of the boom and temperature profile given in Fig. 1 is obtained by multiplying $R_L = 4200$ ft (obtained from the diagram in Fig. 2) by the corresponding value of $\beta = 1.06$ to obtain $R = 4450$ ft. The values of β were also very small for a number of other temperature profiles that were encountered in applications.

Some of the temperature profiles[5] for which the temperature profile factors can be determined in closed form are shown in Fig. 5. For the linear temperature change (Fig. 5a), we have $\beta = 1.23$, and for the step temperature function (Fig. 5b), we get

$$\beta = \pi/4 \sin p\pi \quad (15)$$

where p is the parameter of the step temperature. For $p = 0.29$ and 0.71, the value of $\beta = 1$. For very small and very large values of p, the value of β approaches infinity indicating a significant error of the results calculated, using the linearized temperature profile for this region. The value of β is smaller than zero for $0.29 < p < 0.71$ having a minimum of $\beta = 0.785$ for $p = 0.5$. This example demonstrates that the temperature profile factor β may be a positive or negative number. Although the results obtained for the no-stress and the actual cases do not differ, in general, very much, there are instances for which the discrepancy is significant.

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